Spatial Durbin Mixture Models

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Outline

Motivation

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- Aquaro, Bailey & Pesaran. (2015); LeSage & Chih (2016); Cornwall and Parent (2016): Introduce heterogeneous spatial models predominately in the Spatial Autoregressive framework.

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- Own-partial and cross partial derivatives have a much richer interpretation and are differentiated not only by group but also relative location.
- Conditional group assignment allows for interpretation of unobserved intra- and inter-group dynamics.

Spatial Heterogeneity

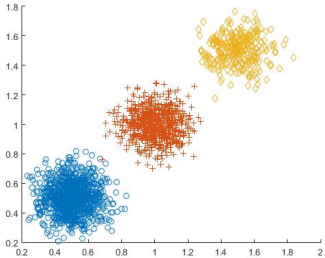

Figure: Spatial Heterogeneity

- Mean varies across **clusters**.
- A form of fixed effects can be implemented by allowing intercept to vary.
- Approach requires delineation of data into smaller clusters which may or may not be feasible.

Spatial Heterogeneity

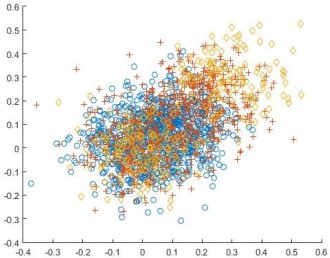

Figure: Spatial Dependence/Mixture

- Mean may or may not vary across **groups** ($g = 1, 2, \ldots, G$)
- Fixed effects may be spurious.
- SAR model overstates spatial dependency.
- Delineation into small **clusters** is nearly impossible with any sort of accuracy

Spatial Durbin Models

Spatial Durbin Model

$$Y = \rho Wy + X\beta + WX\Phi + \epsilon \tag{1}$$

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- W is an $N \times N$ right-stochastic matrix which represents the connectivity structure for the sample.
- ρ is typically positive in homogeneous models but **can** be negative.
- Most other spatial specifications are nested within the structure of the SDM.

Finite Mixture Models

G-component Mixture Model

- A mixture of Gaussian distributions can approximate most other distribution forms.

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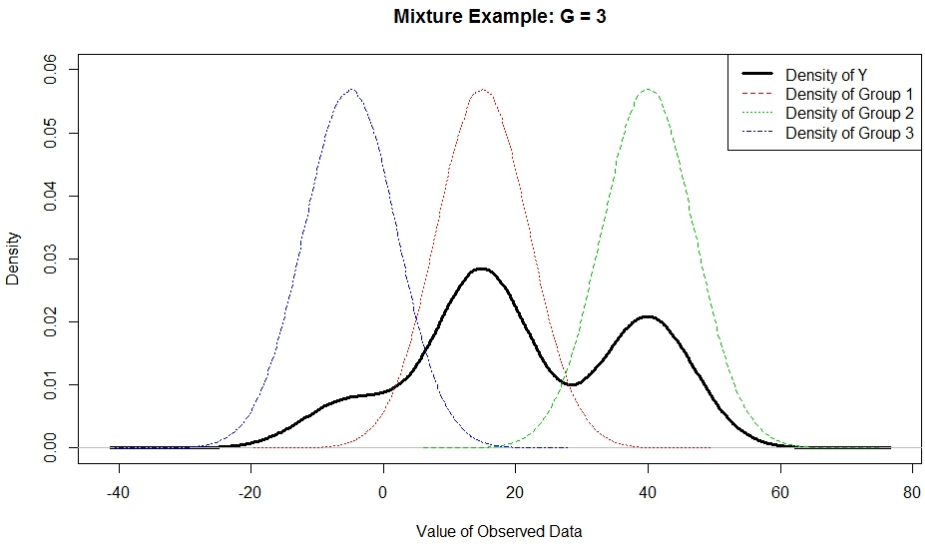
G-component Mixture Model

- A mixture of Gaussian distributions can approximate most other distribution forms.
- Generally can be written as:

$$p(y_i|x_i,\beta,\Sigma,z,\pi) = \sum_{g=1}^{G} \pi_g N(y_i|x_i\beta_g,\sigma_g^2), \quad \sum_{g=1}^{G} \pi_g = 1 \tag{2}$$

$$y_i = \sum_{g=1}^{G} z_{ig}\alpha_g + \sum_{k=1}^{K}\sum_{g=1}^{G} z_{ig}x_i^k\beta_g^k + \sum_{g=1}^{G} \Omega_{gg} z_{ig}\epsilon_i, \quad i = 1,2,\ldots,N \tag{3}$$


Mixture Example: G = 3
Density of Y
Density of Group 1
Density of Group 2
Density of Group 3
Density
0.00
0.01
0.02
0.03
0.04
0.05
0.06
-40
-20
0
20
40
60
80
Value of Observed Data

Spatial Durbin Mixture Model: SDM-M

$$y_i = \sum_{g=1}^{G} z_{ig}\alpha_g + \sum_{g=1}^{G} z_{ig}\rho_g \sum_{j=1}^{N} w_{ij}y_j + \sum_{k=1}^{K}\sum_{g=1}^{G} z_{ig}x_i^k\beta_g^k + \sum_{k=1}^{K}\sum_{g=1}^{G}\sum_{j=1}^{N} w_{ij}z_{ig}x_j^k\phi_g^k + \sum_{g=1}^{G}\Omega_{gg}z_{ig}\epsilon_i \quad (4)$$

Matrix Notation:

$$y = \tilde{\alpha} + \tilde{\Psi}Wy + \tilde{X}B + W\tilde{X}\Phi + \tilde{\epsilon} \quad (5)$$

Definitions:

$$\tilde{\alpha} = z\alpha$$
$$\tilde{X} = (\iota_G' \otimes X) \odot (z \otimes \iota_K')$$

$$\tilde{\Psi} = z\psi,\ \psi = (\rho_1, \ldots, \rho_g)$$
$$\tilde{\epsilon} = (z\Omega^{1/2}) \odot \epsilon$$

Spatial Durbin Error Mixture Model: SDEM-M

$$y_i = \sum_{g=1}^{G} z_{ig}\alpha_g + \sum_{k=1}^{K}\sum_{g=1}^{G} z_{ig}x_i^k\beta_g^k + \sum_{k=1}^{K}\sum_{g=1}^{G}\sum_{j=1}^{N} w_{ij}z_{ig}x_j^k\phi_g^k + \eta_i \tag{6}$$

$$\eta_i = \sum_{g=1}^{G} z_{ig}\lambda_g \sum_{j=1}^{N} w_{ij}\eta_j + \sum_{g=1}^{G} z_{ig}\epsilon_i$$

Matrix notation:

$$y = \tilde{\alpha} + \tilde{X}B + W\tilde{X}\Phi + (I - \tilde{\Psi}W)^{-1}\tilde{\epsilon} \tag{7}$$

The Parameters of Interest

- B, Φ - Each are $KG \times 1$ vectors of coefficients.
- Ω - $G \times 1$ vector of variances.
- ψ - $G \times 1$ vector of scalars indicating strength of spatial dependence.
- ω, z - An $N \times G$ matrix indicating which group each region is in.
- π - A $G \times 1$ vector of group weights over the sample.

Sampling Algorithm

- Set initial values for parameters.
- Expand X to $\tilde{X}$.
- Draw from $p(\bar{B}|\Omega,\psi,z,\pi,x,y) \sim N(D_{\bar{B}}d_{\bar{B}}, D_{\bar{B}})$
- Draw from $p(\Omega|\bar{B},\psi,z,\pi,z,y) \sim IG(c,C)$
- Draw from (M-H Step) $p(\rho_g|\rho_{-g},\Omega,\bar{B},z,\pi,x,y) \propto |I_N - \tilde{\Psi}|exp\big[-\frac{1}{2}e'\Omega^{-1}e\big]$
- Draw from $p(z_i|\Omega,\bar{B},\psi,x,y) \sim MN(1,[\omega_{i1},\ldots,\omega_{iG}])$
- Draw from $p(\pi|\Omega,\bar{B},\psi,z,x,y) \sim D(\alpha+N)$
- Iterate

Partial Derivatives for the SDM-M

For reference the partial derivatives of the SDM are:

$$\frac{\delta y}{\delta x^k} = (I - \rho W)^{-1}(I\beta^k + W\phi^k) \tag{8}$$

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- **Direct Effects:** $n^{-1} tr(S_k(W))$
- **Indirect Effects:** Total - Direct

The "To" and "From" Now Matters

The partial derivative for the SDM-M:

$$\frac{\delta y}{\delta x^k} = (I - \tilde{\Psi} W)^{-1} \left(diag(z\beta_g^k) + diag(z\phi_g^k) W \right) \tag{9}$$

- The partial derivative is still an $N \times N$ matrix.
- Let $M_k(W) = (I - \tilde{\Psi} W)^{-1} \left(diag(z\beta_g^k) + diag(z\phi_g^k) W \right)$.
- Now $n^{-1} \iota_n' c_r \neq n^{-1} r_r \iota_n$.

Relative Location

Defining Interior and Border

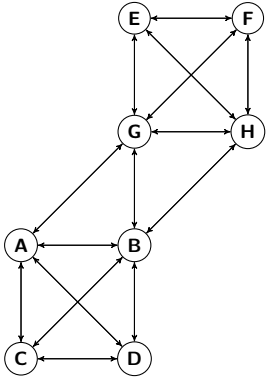
Interior (Λ)

$$i \in \lambda_g \iff j \in g \ \forall \ w_{ij} > 0 \qquad (10)$$

Border (Γ)

$$i \in \gamma_g \ \exists \ j \notin g \ \forall \ w_{ij} > 0 \qquad (11)$$

$$\sum_{g=1}^{G} \lambda_g = \Lambda, \quad \sum_{g=1}^{G} \gamma_g = \Gamma, \quad \Lambda \cup \Gamma = N$$



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- The more regions included as first order neighbors the more isolated a region must be to be considered interior.
- This means that some groups may consist entirely of border agents while others are a mix.

Interior and Border Effects Realized

As a reminder $M_r(W) = (I - \tilde{\Psi} W)^{-1} \left(diag(z\beta_g^k) + diag(z\phi_g^k) W \right)$.

Group Interior Effects:

$$DE_g^\lambda = (n_g^\lambda)^{-1} \sum_{i=1}^{n_g^\lambda} m_{ii} \tag{12}$$

$$SI_g^\lambda = (n_g^\lambda)^{-1} \sum_{i=1}^{n_g^\lambda} \sum_{j=1, j \neq i}^{n_g^\lambda} m_{ij} \tag{13}$$

$$SO_g^\lambda = (n_g^\lambda)^{-1} \sum_{i=1}^{n_g^\lambda} \sum_{i=1, i \neq j}^{n_g^\lambda} m_{ij} \tag{14}$$

Group Border Effects:

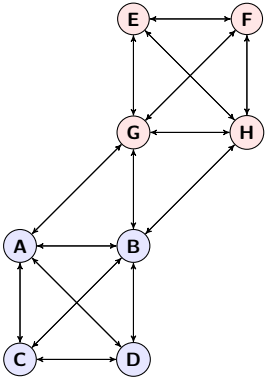
$$DE_g^\gamma = (n_g^\gamma)^{-1} \sum_{i=1}^{n_g^\gamma} m_{ii} \tag{15}$$

$$SI_g^\gamma = (n_g^\gamma)^{-1} \sum_{i=1}^{n_g^\gamma} \sum_{j=1, j \neq i}^{n_g^\gamma} m_{ij} \tag{16}$$

$$SO_g^\gamma = (n_g^\gamma)^{-1} \sum_{i=1}^{n_g^\gamma} \sum_{i=1, i \neq j}^{n_g^\gamma} m_{ij} \tag{17}$$

A Simple Example

- 8 Regions
- 2 Groups
- Contiguity Weight Matrix - Queen
- Group 1:
  - A, B, C, D
  - $\rho_1 = -0.2$
  - $\beta_1^k = 1,\ \phi_1^k = 0$
- Group 2:
  - E, F, G, H
  - $\rho_2 = 0.5$
  - $\beta_2^k = 1,\ \phi_2^k = 0$



The SDM-M Results Table

Table: SDM-M Results

Group	DE^{int}	SI^{int}	SO^{int}	DE^{bor}	SI^{bor}	SO^{bor}
One	1.0096	-0.1693	-0.1658	0.9993	-0.2226	0.1102
Two	1.0949	0.8046	0.5519	1.0668	0.6823	0.5988

The SDM-M reports group level border and interior effects.

- Provides more information than SDM.
- What happens if we vary ρ_2 over the domain while holding ρ_1 constant?

Why interior and border? A visual justification...

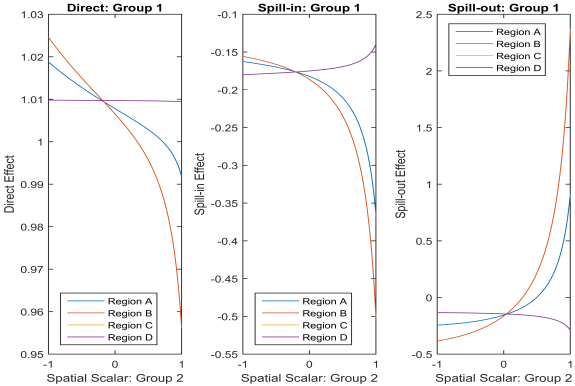
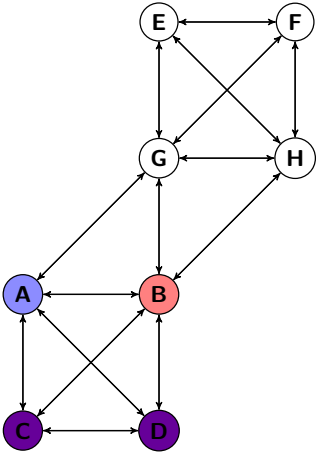


Figure: Group 1 Responses



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- Model: $\log(income) = \tilde{\Psi} W \log(income) + XB + WX\Phi + \epsilon$

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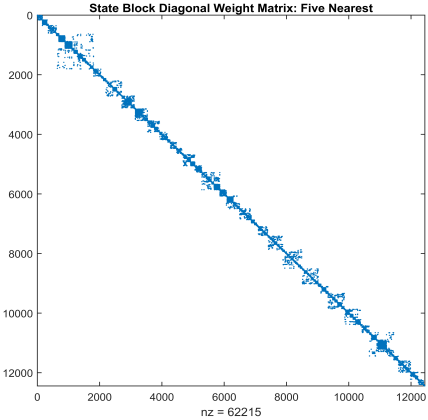
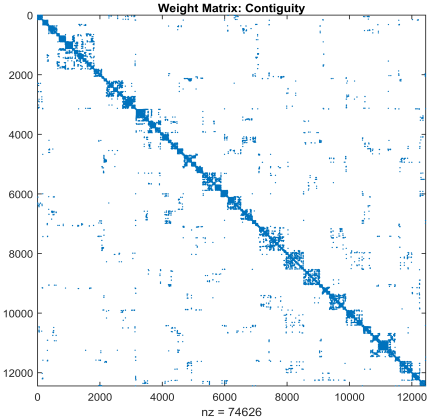
  - Block-diagonal (State) nearest-neighbor (5) weight matrix.

Results Summary

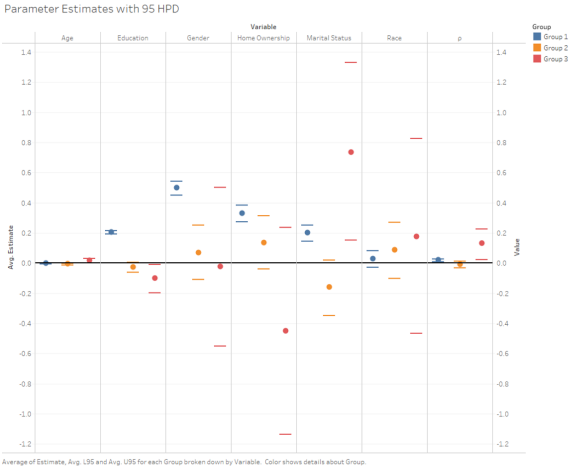
- Three distinct groups emerge from the data.
  - Group 1 - Fully-employed
  - Group 2 - Un-employed
  - Group 3 - Under-employed
- Spill-out effects from education tend to be positive even if the direct effects are not.
- Spill-in effects from education tend to be negative for both those that are un-employed and under-employed.
- Household dynamics become apparent by a *post hoc* analysis of the data under the estimated groupings.
- Estimates are robust to changes in the weight matrix.
  - Nearest Neighbor - 2 through 6.
  - Contiguity - Queen.


State Block Diagonal Weight Matrix: Five Nearest
0
2000
4000
6000
8000
10000
12000
0
2000
4000
6000
8000
10000
12000
nz = 62215

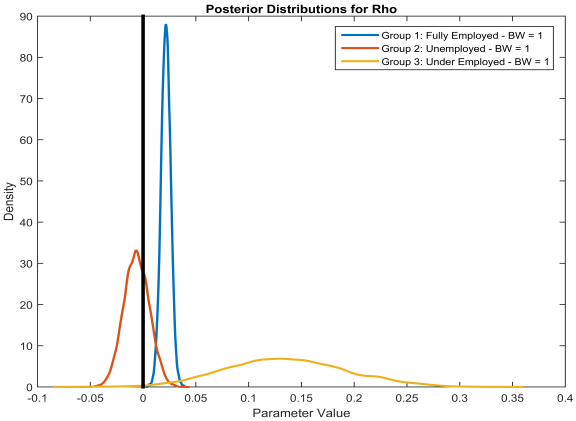

Weight Matrix: Contiguity
0
2000
4000
6000
8000
10000
12000
0
2000
4000
6000
8000
10000
12000
nz = 74626

Parameter Estimates

- All hypothesis are examined under 95% HPD.
- Many of the spatially lagged characteristics have significant estimates (Φ).
- Three Groups
  - Group One (*Fully-Employed*) - 80.4%
  - Group Two (*Unemployed*) - 14.3%
  - Group Three (*Under-Employed*) - 5.3%



Posteriors for Ψ



- Odds against (Mills, WP) $\rho_1 = 0 \approx 346,000 : 1$
- Odds against (Mills, WP) $\rho_2 = 0 \approx 1.19 : 1$
- Odds against (Mills, WP) $\rho_3 = 0 \approx 21.5 : 1$

Select Results: Education

- Obtaining additional human capital has large and positive impacts on income (Psacharopoulos, 1994; Montenegro & Patrinos, 2013; etc.)
- Agents in group 2 experience a negative spill-in when their neighbors acquire additional human capital.
- Agents in group 3 experience downward pressure on wages as a result of obtaining additional human capital.

		Estimate	Std. Dev.	Lower 95	Upper 95
Group One	Interior Direct	0.0716**	0.0026	0.0666	0.0766
	Interior Spill-in	0.1307**	0.0048	0.1213	0.1397
	Interior Spill-out	0.1225**	0.0046	0.1133	0.1311
	Border Direct	0.1356**	0.0042	0.1272	0.1437
	Border Spill-in	0.2474**	0.0062	0.2348	0.2596
	Border Spill-out	0.1742**	0.0074	0.1598	0.1881
Group Two	Interior Direct	-	-	-	-
	Interior Spill-in	-	-	-	-
	Interior Spill-out	-	-	-	-
	Border Direct	-0.0254	0.0167	-0.0595	0.0066
	Border Spill-in	-0.2149**	0.0233	-0.2616	-0.1723
	Border Spill-out	0.2876**	0.0078	0.2727	0.3031
Group Three	Interior Direct	-	-	-	-
	Interior Spill-in	-	-	-	-
	Interior Spill-out	-	-	-	-
	Border Direct	-0.0957**	0.0495	-0.1920	-0.0008
	Border Spill-in	0.4219**	0.0875	0.2423	0.5918
	Border Spill-out	0.3022**	0.0101	0.2830	0.3217

A Quick Summary

- The spatial mixture class of models fills a gap between traditional spatial models (see LeSage & Pace 2009) and more recent developments in heterogeneous models (see Aquaro, et al., 2015 and LeSage & Chih, 2016).

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- The SDM-M and SDEM-M nest many other functional forms as special cases.
- The focus on group and locational dynamics gives avenues for model exploration that previously were not available.

Thank you!

Parameter Estimates

	Group 1			Group 2			Group 3		
	Estimate	L-95 HPD	U-95 HPD	Estimate	L-95 HPD	U-95 HPD	Estimate	L-95 HPD	U-95 HPD
β_{Age}	0.0009	-0.0006	0.0024	-0.0055	-0.0111	0.0004	0.0195**	0.0037	0.0354
$\beta_{Education}$	0.2072**	0.1979	0.2167	-0.0255	-0.0591	0.0085	-0.0981**	-0.1920	-0.0050
β_{Gender}	0.4993**	0.4541	0.5453	0.0692	-0.1061	0.2564	-0.0228	-0.5471	0.5042
β_{Race}	0.0295	-0.0253	0.0852	0.0863	-0.0975	0.2743	0.1751	-0.4633	0.8289
$\beta_{MaritalStatus}$	0.2016**	0.1496	0.2547	-0.1575	-0.3447	0.0230	0.7359**	0.1578	1.3355
$\beta_{HomeOwnership}$	0.3312**	0.2763	0.3874	0.1363	-0.0363	0.3170	-0.4495	-1.1340	0.2422
ϕ_{Age}	0.0406**	0.0378	0.0435	-0.0256**	-0.0348	-0.0163	0.0595**	0.0289	0.0890
$\phi_{Education}$	0.3542**	0.3409	0.3671	-0.2079**	-0.2530	-0.1651	0.2655**	0.1091	0.4212
ϕ_{Gender}	0.2834**	0.1735	0.3940	-0.2243	-0.5819	0.1351	0.2600	-0.9556	1.4924
ϕ_{Race}	-0.4437**	-0.5279	-0.3621	0.2585	-0.0235	0.5417	-0.6528	-1.6213	0.3406
$\phi_{MaritalStatus}$	0.6616**	0.5490	0.7747	-0.3833**	-0.7696	-0.0160	1.7938**	0.5373	3.0427
$\phi_{HomeOwnership}$	-0.6111**	-0.7232	-0.5027	0.3602**	0.0089	0.7047	-0.7392	-1.9791	0.5055
ρ	0.0219**	0.0123	0.0309	-0.0061	-0.0291	0.0172	0.1331**	0.0254	0.2288
σ^2	1.0631	0.9788	1.1628	1.3474	0.7116	2.0955	5.2154	4.1524	6.4981
π	0.8040	0.7925	0.8148	0.1429	0.1368	0.1492	0.0531	0.0435	0.0628
Nobs	9995			1782			666		

Partial Effects Summaries

		Group 1 - Fully Employed				Group 2 - Unemployed				Group 3 - Under Employed			
		Estimate	std	l95	u95	Estimate	std	l95	u95	Estimate	std	l95	u95
Age	Interior Direct	0.0004	0.0002	0.0008	-0.0001	0.0000	-	-	-	0.0000	-	-	-
	Interior Spill-in	0.0225**	0.0018	0.0193	0.0264	0.0000	-	-	-	0.0000	-	-	-
	Interior Spill-out	0.0220**	0.0017	0.0187	0.0257	0.0000	-	-	-	0.0000	-	-	-
	Border Direct	0.0007	0.0004	-0.0003	0.0016	-0.0054	0.0029	-0.0109	0.0004	0.0232**	0.0084	0.0067	0.0399
	Border Spill-in	0.0426**	0.0032	0.0368	0.0492	-0.0343**	0.0122	-0.0574	-0.0100	0.2184**	0.0771	0.0746	0.3425
	Border Spill-out	0.0372**	0.0050	0.0279	0.0480	0.0571**	0.0059	0.0702	0.0460	0.0613**	0.0070	0.0482	0.0762
Education	Interior Direct	0.0716**	0.0026	0.0666	0.0766	0.0000	-	-	-	0.0000	-	-	-
	Interior Spill-in	0.1307**	0.0048	0.1213	0.1397	0.0000	-	-	-	0.0000	-	-	-
	Interior Spill-out	0.1225**	0.0046	0.1133	0.1311	0.0000	-	-	-	0.0000	-	-	-
	Border Direct	0.1356**	0.0042	0.1272	0.1437	-0.0254	0.0167	-0.0595	0.0066	-0.0957**	0.0495	-0.1920	-0.0008
	Border Spill-in	0.2474**	0.0062	0.2348	0.2596	-0.2149**	0.0233	-0.1723	-0.2616	0.4219**	0.0875	0.2423	0.5918
	Border Spill-out	0.1742**	0.0074	0.1598	0.1881	0.2876**	0.0078	0.2727	0.3031	0.3022**	0.0101	0.2830	0.3217
Gender	Interior Direct	0.1729**	0.0101	0.1531	0.1924	0.0000	-	-	-	0.0000	-	-	-
	Interior Spill-in	0.1060**	0.0195	0.0684	0.1453	0.0000	-	-	-	0.0000	-	-	-
	Interior Spill-out	0.0987**	0.0185	0.0637	0.1365	0.0000	-	-	-	0.0000	-	-	-
	Border Direct	0.3272**	0.0161	0.2963	0.3593	0.0674	0.0899	-0.1076	0.2460	-0.0256	0.2642	-0.5530	0.5035
	Border Spill-in	0.2005**	0.0366	0.1313	0.2742	-0.2339	0.1826	-0.5961	0.1188	0.3905	0.6322	-0.8731	1.5896
	Border Spill-out	0.1360**	0.0445	0.0506	0.2246	0.2264**	0.0582	0.1168	0.3389	0.2402**	0.0605	0.1258	0.3603

Partial Effects Summaries: Continued

		Group 1 - Fully Employed				Group 2 - Unemployed				Group 3 - Under Employed			
		Estimate	std	l95	u95	Estimate	std	l95	u95	Estimate	std	l95	u95
Race	Interior Direct	0.0105	0.0095	-0.0071	0.0300	0.0000	-	-	-	0.0000	-	-	-
	Interior Spill-in	-0.1456**	0.0152	-0.1757	-0.1162	0.0000	-	-	-	0.0000	-	-	-
	Interior Spill-out	-0.1361**	0.0145	-0.1643	-0.1081	0.0000	-	-	-	0.0000	-	-	-
	Border Direct	0.0198	0.0180	-0.0134	0.0572	0.0859	0.0938	-0.0964	0.2795	0.1897	0.3465	-0.4719	0.8791
	Border Spill-in	-0.2755**	0.0276	-0.3322	-0.2220	0.2504	0.1436	-0.0231	0.5353	-0.4946	0.4924	0.4566	-1.4411
	Border Spill-out	-0.1940**	0.0348	-0.2645	-0.1250	-0.3199**	0.0447	-0.4076	-0.2322	-0.3367**	0.0468	-0.4288	-0.2464
Marital Status	Interior Direct	0.0704**	0.0098	0.0525	0.0900	0.0000	-	-	-	0.0000	-	-	-
	Interior Spill-in	0.2354**	0.0227	0.1914	0.2792	0.0000	-	-	-	0.0000	-	-	-
	Interior Spill-out	0.2259**	0.0214	0.1840	0.2672	0.0000	-	-	-	0.0000	-	-	-
	Border Direct	0.1330**	0.0180	0.0997	0.1691	-0.1586	0.0956	-0.3580	0.0286	0.7444**	0.3152	0.1559	1.3406
	Border Spill-in	0.4429**	0.0392	0.3679	0.5156	-0.3927**	0.1929	-0.7824	-0.0236	1.8811**	0.6586	0.6214	3.1805
	Border Spill-out	0.3652**	0.0494	0.2646	0.4563	0.5614**	0.0631	0.4317	0.6813	0.6259**	0.0681	0.4880	0.7550
Home Own	Interior Direct	0.1145**	0.0105	0.0942	0.1355	0.0000	-	-	-	0.0000	-	-	-
	Interior Spill-in	-0.2034**	0.0204	-0.2448	-0.1627	0.0000	-	-	-	0.0000	-	-	-
	Interior Spill-out	-0.1896**	0.0195	-0.2297	-0.1517	0.0000	-	-	-	0.0000	-	-	-
	Border Direct	0.2167**	0.0186	0.1792	0.2535	0.1331	0.0922	-0.0454	0.3115	-0.4598	0.3570	-1.1588	0.2719
	Border Spill-in	-0.3847**	0.0358	-0.3128	-0.4586	0.3565**	0.1756	0.0171	0.7130	-0.5776	0.6262	-1.8065	0.6859
	Border Spill-out	-0.2647**	0.0446	-0.3503	-0.1827	-0.4404**	0.0579	-0.5547	-0.3303	-0.4627**	0.0613	-0.5862	-0.3455

Conditional Distributions: SDM-M

1. Set initial values for parameters.
2. Expand X to $\tilde{X}$.
   1. $\tilde{X} = (\iota_G' \otimes X) \odot (z \otimes \iota_K')$
3. $p(\bar{B}|\Omega, \psi, z, \pi, x, y) \sim N(D_{\bar{B}} d_{\bar{B}}, D_{\bar{B}})$
   1. $D_{\bar{B}} = (\tilde{\tilde{X}}' \Omega^{-1} \tilde{\tilde{X}} + V_{\bar{B}})$
   2. $d_{\bar{B}} = \tilde{\tilde{X}}' \Omega^{-1} \tilde{y} + V_{\bar{B}} \bar{B}_0$
   3. $\tilde{y} = (I_N - \tilde{\Psi} W) y$
4. $p(\Omega|\bar{B}, \psi, z, \pi, z, y) \sim IG(c, C)$
   1. $C = a + \frac{N}{2}$
   2. $c = b + \frac{1}{2} e' e$
   3. $e = \tilde{y} - \tilde{\tilde{X}} \bar{B}$

- $p(\rho_g|\rho_{-g}, \Omega, \bar{B}, z, \pi, x, y) \propto |I_N - \tilde{\Psi}| exp\left[\frac{-1}{2} e' \Omega^{-1} e\right]$
- $p(z_i|\Omega, \bar{B}, \psi, x, y) \sim MN(1, [\omega_{i1}, \omega_{i2}, \ldots, \omega_{iG}])$
  - $\omega_{ig} = \frac{q_{ig}}{\sum_{g=1}^{G} q_{ig}}$
  - $q_{ig} = (2\pi\sigma_g^2)^{-1/2} \exp\left[-\frac{1}{2\sigma_g^2}\left(y_i - \rho_g \sum_{j=1}^{N} w_{ij} y_j - x_i \beta_g - \phi_g \sum_{j=1}^{N} w_{ij} x_i\right)^2\right]$
- $p(\pi|\Omega, \bar{B}, \psi, x, y, z) \sim D(\alpha + N)$